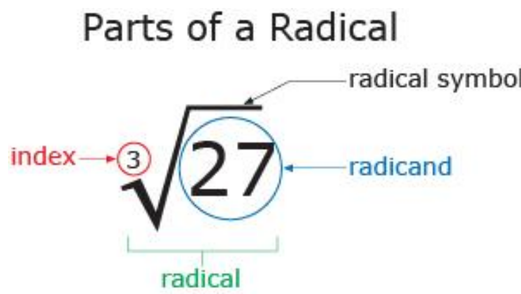
Pre-AP Algebra II
Notes Day #[redacted] 98
Solving Radical Equations



SOLVING RADICAL EQUATIONS

Radical equations include radical expressions. We can solve a radical equation by raising each side of the equation to a power.

Steps for Solving Radical Equations

Step 1: Isolate the radical on one side of the equation.
Step 2: Raise each side of the equation to a power equal to the index of the radical to eliminate the radical.
Step 3: Solve the resulting polynomial equation. Check your results.

(Steps 1 and 2 may need to be repeated if there are multiple radicals in the equation.)

When solving radical equations, the result may be a number that does not satisfy the original equation. Such a number is called an extraneous solution.

Directions: Solve the following equations. Check for extraneous solutions.

Ex. 1: $\sqrt{x-1}=3$

$(\sqrt{x-1})^2 = (3)^2$

$x-1 = 9$

$+1 \quad +1$

$\boxed{x=10}$

CHECK:

$\sqrt{x-1} = 3$

$\sqrt{10-1} = 3$

$\sqrt{9} = 3$

$3 = 3$ ✓

Ex. 2: $2+\sqrt{3x-2}=6$

$-2 \quad -2$

$\sqrt{3x-2} = 4$

$(\sqrt{3x-2})^2 = (4)^2$

$3x-2 = 16$

$+2 \quad +2$

$\frac{3x}{3} = \frac{18}{3}$

$\boxed{x = \frac{18}{3}}$

CHECK:

$2+\sqrt{3x-2} = 6$

$2+\sqrt{3(\frac{18}{3})-2} = 6$

$2+\sqrt{18-2} = 6$

$2+\sqrt{16} = 6$

$2+4 = 6$

$6 = 6$ ✓

Directions: **Solve the following equations. Check for extraneous solutions.**

Ex. 3: $\sqrt{x-3}+5=x$

$$-5 \quad -5$$
$$\sqrt{x-3} = x-5$$
$$(\sqrt{x-3})^2 = (x-5)^2$$
$$x-3 = (x-5)(x-5)$$
$$x-3 = x^2-5x-5x+25$$
$$x-3 = x^2-10x+25$$
$$+3 \qquad +3$$
$$x = x^2-10x+28$$
$$-x \qquad -x$$
$$0 = x^2-11x+28$$
$$a\cdot c = 28$$
$$b=-11$$
$$0 = (x-7)(x-4) \quad f: -7, -4$$
$$x-7=0 \qquad x-4=0$$
$$+7 \ +7 \qquad +4 \ +4$$
$$\boxed{x=7} \qquad \cancel{\boxed{x=4}}$$

CHECK:

$\sqrt{x-3}+5=x$

Check $x=7$

$\sqrt{7-3}+5=7$

$\sqrt{4}+5=7$

$2+5=7$

$7=7$ ✓

$\sqrt{x-3}+5=x$

Check $x=4$

$\sqrt{4-3}+5=4$

$\sqrt{1}+5=4$

$1+5=4$

$6=4$ ✗

Ex. 4: $\sqrt{x-1}=\sqrt{x+4}$

$$(\sqrt{x-1})^2 = (\sqrt{x+4})^2$$
$$x-1 = x+4$$
$$-x \qquad -x$$
$$-1 = 4 \quad ✗$$

$\boxed{\text{No Solution}}$

CHECK:

Ex. 5: $\sqrt{x+15}=5+\sqrt{x}$

$$(\sqrt{x+15})^2 = (5+\sqrt{x})^2$$
$$x+15 = (5+\sqrt{x})(5+\sqrt{x})$$
$$x+15 = 25+5\sqrt{x}+5\sqrt{x}+x$$
$$x+15 = 25+10\sqrt{x}+x$$
$$-x \ -25 \qquad -25 \qquad -x$$
$$\frac{-10}{10} = \frac{10\sqrt{x}}{10}$$
$$(-1)^2 = (\sqrt{x})^2$$
$$\cancel{\boxed{1=x}}$$

CHECK:

$\sqrt{x+15}=5+\sqrt{x}$

Check $x=1$

$\sqrt{1+15}=5+\sqrt{1}$

$\sqrt{16}=5+1$

$4=6$ ✗

We can apply the methods used to solve square root equations to solving equations with roots of any index. To undo an *n*th root, raise to the *n*th power.

Ex. 6: $4\sqrt[4]{3x+6}-12=0$

$$+12 \quad +12$$
$$\frac{4\sqrt[4]{3x+6}}{4} = \frac{12}{4}$$
$$\sqrt[4]{3x+6} = 3$$
$$(\sqrt[4]{3x+6})^4 = (3)^4$$
$$3x+6 = 81$$
$$-6 \quad -6$$
$$\frac{3x}{3} = \frac{75}{3}$$
$$\boxed{x=25}$$

CHECK:

$4\sqrt[4]{3x+6}=12$

Check $x=25$

$4\sqrt[4]{3(25)+6}=12$

$4\sqrt[4]{75+6}=12$

$4\sqrt[4]{81}=12$

$4(3)=12$

$12=12$ ✓