Pre-AP Algebra II
Notes Day #[redacted] 101
Simplifying Radical Expressions

Directions: Simplify the radical expressions.

Ex. 1: $\sqrt{81x^6z^{14}}$

$$= \boxed{9|x^3z^7|}$$

Ex. 2: $\sqrt[4]{81x^5y^{12}z^{10}}$

$$= \boxed{9x|y^3|z^2\sqrt[4]{xz^2}}$$

The Quotient Property of Radicals is another property used to simplify radicals.
For any real numbers a and $b \neq 0$ and any integer $n > 1$,

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$ **if all roots are defined.**

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt{9} \text{ or } 3 \qquad \sqrt[3]{\frac{x^6}{8}} = \frac{\sqrt[3]{x^6}}{\sqrt[3]{8}} = \frac{x^2}{2} \text{ or } \frac{1}{2}x^2$$

To eliminate radicals from a denominator or fractions from a radicand, we can use a process called rationalizing the denominator. To rationalize a denominator, multiply the numerator and denominator by a quantity so that the radicand has an exact root.

If the Denominator is:	Multiply the numerator and denominator by:	Examples
$\sqrt{b}$	$\sqrt{b}$	$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
$\sqrt[a]{b^x}$	$\sqrt[a]{b^{n-x}}$	$\frac{5}{\sqrt[3]{2}} = \frac{5}{\sqrt[3]{2}} \bullet \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} = \frac{5\sqrt[3]{4}}{2}$

Directions: Simplify.

Ex. 3: $\frac{\sqrt{x^9}}{\sqrt{y^5}} \cdot \frac{\sqrt{y}}{\sqrt{y}}$

$$= \frac{\sqrt{x^9y}}{\sqrt{y^6}}$$

$$= \boxed{\frac{x^4\sqrt{xy}}{|y^3|}}$$

Ex. 4: $\sqrt[5]{\frac{3}{4y}} = \frac{\sqrt[5]{3}}{\sqrt[5]{2^2y}} \cdot \frac{\sqrt[5]{2^3y^4}}{\sqrt[5]{2^3y^4}}$

$$= \frac{\sqrt[5]{3(2^3)y^4}}{\sqrt[5]{2^5y^5}}$$

$$= \frac{\sqrt[5]{3(8)y^4}}{2y}$$

$$= \boxed{\frac{\sqrt[5]{24y^4}}{2y}}$$

Rules used to simplify radicals.

A radical expression is in simplified form when the following conditions are met.

- **The index n is as small as possible**
- **The radicand contains no factors (other than 1) that are *n*th powers of an integer or polynomial.**
- **The radicand contains no fractions.**
- **No radicals appear in a denominator.**

Operations with Radicals

We can use the Product and Quotient Properties to multiply and divide some radicals.

Directions: **Simplify the radical expressions.**

Ex. 5: $6\sqrt{8x^3y^5} \bullet 4\sqrt{2xy^3}$

$= 24\sqrt{16x^4y^8}$

$= 24(4)x^2y^4$

$= \boxed{96x^2y^4}$

Ex. 6: $2\sqrt[4]{8x^3y^2} \bullet 3\sqrt[4]{2x^5y^2}$

$= 6\sqrt[4]{16x^8y^4}$

$= 6(2)x^2|y^1|$ → even-even-odd rule

$= \boxed{16x^2|y^1|}$

Radicals can be added and subtracted in the same manner as monomials. In order to add or subtract, the radicals must be like terms. Radicals are like radical expressions if <u>both</u> the index and the radicand are identical.

Directions: **Simplify.**

Ex. 7: $4\sqrt{8} + 3\sqrt{50}$

$4(\sqrt{4})(\sqrt{2}) + 3(\sqrt{25})(\sqrt{2})$

$4(2)\sqrt{2} + 3(5)\sqrt{2}$

$8\sqrt{2} + 15\sqrt{2}$

$\boxed{23\sqrt{2}}$

Ex. 8: $5\sqrt{12} + 2\sqrt{27} - \sqrt{128}$

$5(\sqrt{4})(\sqrt{3}) + 2(\sqrt{9})(\sqrt{3}) - (\sqrt{64})(\sqrt{2})$

$5(2)\sqrt{3} + 2(3)(\sqrt{3}) - 8\sqrt{2}$

$10\sqrt{3} + 6\sqrt{3} - 8\sqrt{2}$

$\boxed{16\sqrt{3} - 8\sqrt{2}}$

Ex. 9: $(6\sqrt{3} - 5)(2\sqrt{5} + 4\sqrt{2})$

$\boxed{12\sqrt{15} + 24\sqrt{6} - 10\sqrt{5} - 20\sqrt{2}}$

Ex. 10: $(7\sqrt{2} - 3\sqrt{3})(7\sqrt{2} + 3\sqrt{3})$

$49\sqrt{4} + \cancel{21\sqrt{6}} - \cancel{21\sqrt{6}} - 9\sqrt{9}$

$49(2) - 9(3)$

$98 - 27$

$\boxed{71}$