Pre-AP Algebra II
Notes Day #[redacted] 100
Rules of Exponents and Radicals

For $a > 0$, $b > 0$, and all values of m and n, these following properties are true:

Product Property of Exponents
$\boldsymbol{a^m \cdot a^n = a^{m+n}}$

Ex. 1: $x^6 \bullet x^7 = x^{6+7} = \boxed{x^{13}}$

Quotient Property of Exponents
$\boldsymbol{\dfrac{a^m}{a^n} = a^{m-n}}$

Ex. 2: $\dfrac{x^7}{x} = x^{7-1} = \boxed{x^6}$

Power of a Power Property
$\boldsymbol{(a^m)^n = a^{mn}}$

Ex. 3: $(x^6)^3 = x^{6\cdot 3} = \boxed{x^{18}}$

Power of a Product Property
$\boldsymbol{(ab)^m = a^m b^m}$

Ex. 4: $(3x^3y)^2 = (3^2x^{3\cdot 2}y^{1\cdot 2})$
$= \boxed{9x^6y^2}$

Power of a Quotient Property
$\boldsymbol{\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}}$

Ex. 5: $\left(\dfrac{3x^2}{y^3}\right)^4 = \dfrac{3^4x^{2\cdot 4}}{y^{3\cdot 4}}$
$= \boxed{\dfrac{81x^8}{y^{12}}}$

Definition of Negative Exponents
$\boldsymbol{a^{-n} = \dfrac{1}{a^n}}$ ***or*** $\boldsymbol{\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n}$

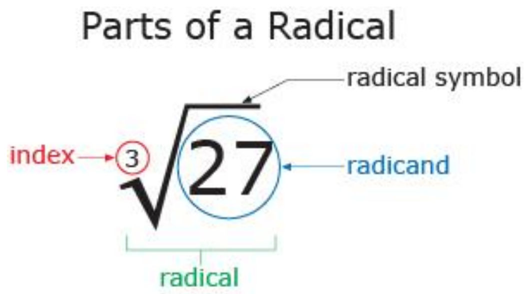
Ex. 6: $\dfrac{x^{-3}}{1} = \boxed{\dfrac{1}{x^3}}$

$\dfrac{1}{y^{-2}} = \boxed{y^2}$

Zero Exponents
$\boldsymbol{a^0 = 1}$

Power Property of Equality
If $a = b$, then $a^n = b^n$

Common Base Property of Equality
If $a^n = a^m$, then $n = m$



RELATIONSHIP BETWEEN RADICALS AND EXPONENTS

If m and n are positive integers, and $\sqrt[n]{a}$ is a real number, then

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$\text{and } a^{\frac{m}{n}} = \sqrt[n]{a^m} \text{ or } a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

Directions: **Write in radical form.**

Ex. 7: $x^{\frac{1}{2}} = \boxed{\sqrt{x}}$

Ex. 8: $x^{\frac{3}{5}} = \boxed{\sqrt[5]{x^3}}$

Directions: **Write in exponential form.**

Ex. 9: $\sqrt[3]{x} = \boxed{x^{\frac{1}{3}}}$

Ex. 10: $\sqrt[5]{8y^2}$

$\boxed{(8y^2)^{\frac{1}{5}} \text{ or } 8^{\frac{1}{5}} y^{\frac{2}{5}}}$

Directions: **Evaluate the following expressions.**

Ex. 11: $27^{\frac{1}{3}} = \sqrt[3]{27}$

$= \boxed{3}$

Ex. 12: $\frac{25^{-\frac{1}{2}}}{1} = \frac{1}{25^{\frac{1}{2}}}$

$= \frac{1}{\sqrt{25}}$

$= \boxed{\frac{1}{5}}$

Ex. 13: $(-32)^{\frac{3}{5}} = \sqrt[5]{(-32)^3}$ → can apply either the fifth root or third power

$= (-2)^3$

$= \boxed{-8}$

Directions: **Simplify:**

Ex. 14: $x^{\frac{2}{3}} \bullet x^{\frac{4}{9}} = x^{\frac{2}{3}+\frac{4}{9}}$

$= x^{\frac{6}{9}+\frac{4}{9}}$

$= \boxed{x^{\frac{10}{9}}}$

Ex. 15: $\left(\dfrac{x^{-\frac{1}{4}}}{y^{-\frac{3}{4}}}\right)^{12} = \left(\dfrac{y^{\frac{3}{4}}}{x^{\frac{1}{4}}}\right)^{12}$

$= \dfrac{y^{\frac{36}{4}}}{x^{\frac{12}{4}}}$

$= \boxed{\dfrac{y^9}{x^3}}$

Simplifying Radicals

The properties we have used to simplify radical expressions involving square roots also hold true for expressions involving *n*th roots.

Product Property of Radicals

For any real numbers *a* and *b* and any integer $n > 1, \sqrt[n]{ab} = \sqrt[n]{a} \bullet \sqrt[n]{b}$ **, if *n* is even and *a* and *b* are both nonnegative or if *n* is odd.**

$\sqrt{2} \bullet \sqrt{8} = \sqrt{16}$ **or** 4**, and** $\sqrt[3]{3} \bullet \sqrt[3]{9} = \sqrt[3]{27}$ **or** 3

In order for a radical to be in simplest form, the radicand must contain no factors that are *n*th powers of an integer or polynomial.

Directions: **Simplify.**

Ex. 16: $\sqrt{81x^6z^{14}} = (\sqrt{81})(\sqrt{x^6})(\sqrt{z^{14}})$

$= \boxed{9|x^3 z^7|}$

↳ even-even-odd rule: If both the exponent and root are even and the answer is an odd exponent, then absolute value bars are placed around the odd exponent

Ex. 17: $\sqrt[3]{27y^{12}z^7} = (\sqrt[3]{27})(\sqrt[3]{y^{12}})(\sqrt[3]{z^7})$

$= \boxed{3y^4z^2\sqrt[3]{z^1}}$

↳ rule does not apply to odd roots

Ex. 18: $\sqrt[4]{81x^5y^{12}z^{10}}$

$= \boxed{3|xy^3z^5|\sqrt[4]{x}}$