

Pre-AP Algebra II
Notes Day # 95
Graph Symmetries

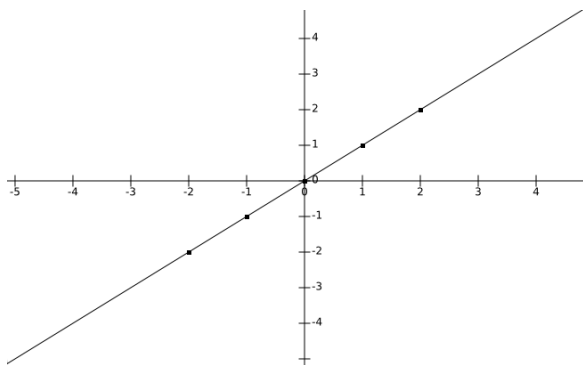
There is an algebraic way to represent symmetry for the points on a graph and how to perform specific types of transformations. The table below demonstrates the transformation from the original point on the graph to its symmetric point for each type of symmetry.

Original $f(x)$	Reflection over y-axis	Reflection over x-axis	180° Rotation	Inverse $f^{-1}(x)$	90° Rotation
(x, y)	$(-x, y)$	$(x, -y)$	$(-x, -y)$	(y, x)	$(-y, x)$ or $(y, -x)$
$(2, 4)$	$(-2, 4)$	$(2, -4)$	$(-2, -4)$	$(4, 2)$	$(-4, 2)$ or $(4, -2)$
$(3, 9)$	$(-3, 9)$	$(3, -9)$	$(-3, -9)$	$(9, 3)$	$(-9, 3)$ or $(9, -3)$
$(-2, 8)$	$(2, 8)$	$(-2, -8)$	$(2, -8)$	$(8, -2)$	$(-8, -2)$ or $(8, 2)$

When doing this for a function, all of the symmetry points must be on the original graph to say that the function exhibits that type of symmetry. If there is a symmetry point not on the original graph, we say that the function does not have this symmetry.

Ex. 1: $y = x$ [Linear parent function]

Original Graph, $f(x)$



x	y
-2	-2
-1	-1
0	0
1	1
2	2

Even Function \times

Odd Function \checkmark

Reflection over y-axis	Reflection over x-axis	180° Rotation	Inverse $f^{-1}(x)$	90° Rotation
$(2, -2)$	$(-2, 2)$	$(2, 2)$	$(-2, -2)$	$(2, -2)$ $(-2, 2)$
$(1, -1)$	$(-1, 1)$	$(1, 1)$	$(-1, -1)$	$(1, -1)$ $(-1, 1)$
$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$ $(0, 0)$
$(-1, 1)$	$(1, -1)$	$(-1, -1)$	$(1, 1)$	$(-1, 1)$ $(1, -1)$
$(-2, 2)$	$(2, -2)$	$(-2, -2)$	$(2, 2)$	$(-2, 2)$ $(2, -2)$

Are all of the points for the y-axis reflection on the original graph? Yes **(No)**

Then the function has no y-axis symmetry

Are all of the points for the x-axis reflection on the original graph? Yes **(No)**

Then the function has no x-axis symmetry

Are all of the points for the 180° rotation on the original graph? **(Yes)** No

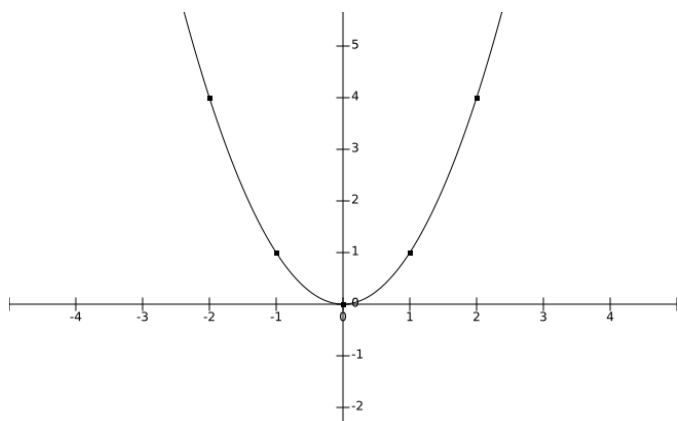
Then the function has 180° rotational symmetry about the origin

Are all of the points in one of the columns for the 90° rotation on the original graph? Yes **(No)**

Then the function has no 90° rotational symmetry

Ex. 2: $y = x^2$ [Quadratic parent function]

Original Graph, $f(x)$



x	y
-2	4
-1	1
0	0
1	1
2	4

Even Function ✓

Odd Function ✗

Reflection over y-axis	Reflection over x-axis	180° Rotation	Inverse $f^{-1}(x)$	90° Rotation	
(2, 4)	(-2, -4)	(2, -4)	(4, -2)	(-4, -2)	(4, 2)
(1, 1)	(-1, -1)	(1, -1)	(1, -1)	(-1, -1)	(1, 1)
(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
(-1, 1)	(1, -1)	(-1, -1)	(1, 1)	(-1, 1)	(1, -1)
(-2, 4)	(2, -4)	(-2, -4)	(4, 2)	(-4, 2)	(4, -2)

Are all of the points for the y-axis reflection on the original graph? Yes No

Then the function has y-axis symmetry

Are all of the points for the x-axis reflection on the original graph? Yes Yes No

Then the function has no x-axis symmetry

Are all of the points for the 180° rotation on the original graph? Yes Yes No

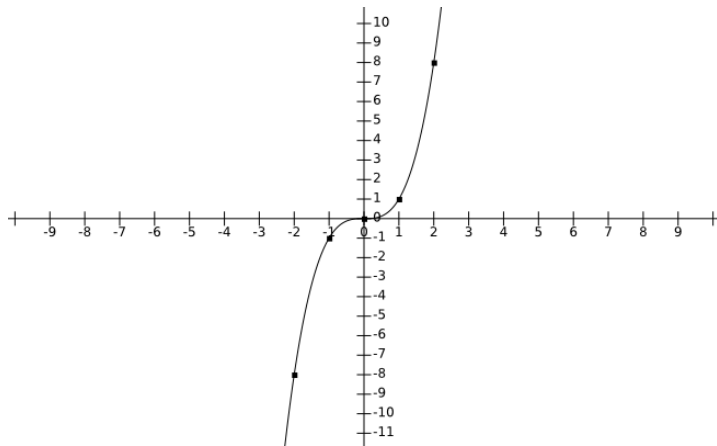
Then the function has no 180° rotational symmetry about the origin

Are all of the points in one of the columns for the 90° rotation on the original graph? Yes Yes No

Then the function has no 90° rotational symmetry

Ex 3: $y = x^3$ [Cube parent function]

Original Graph, $f(x)$



x	y
-2	-8
-1	-1
0	0
1	1
2	8

Even Function **X**

Odd Function **✓**

Reflection over y-axis	Reflection over x-axis	180° Rotation	Inverse $f^{-1}(x)$	90° Rotation	
(2, -8)	(-2, 8)	(2, 8)	(-8, -2)	(8, -2)	(-8, 2)
(1, -1)	(-1, 1)	(1, 1)	(-1, -1)	(1, -1)	(-1, 1)
(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
(-1, 1)	(1, -1)	(-1, -1)	(1, 1)	(-1, 1)	(1, -1)
(-2, 8)	(2, -8)	(-2, -8)	(8, 2)	(-8, 2)	(8, -2)

Are all of the points for the y-axis reflection on the original graph? Yes **(No)**

Then the function has no y-axis symmetry

Are all of the points for the x-axis reflection on the original graph? Yes **(No)**

Then the function has no x-axis symmetry

Are all of the points for the 180° rotation on the original graph? **(Yes)** No

Then the function has 180° rotational symmetry about the origin

Are all of the points in one of the columns for the 90° rotation on the original graph? Yes **(No)**

Then the function has no 90° rotational symmetry