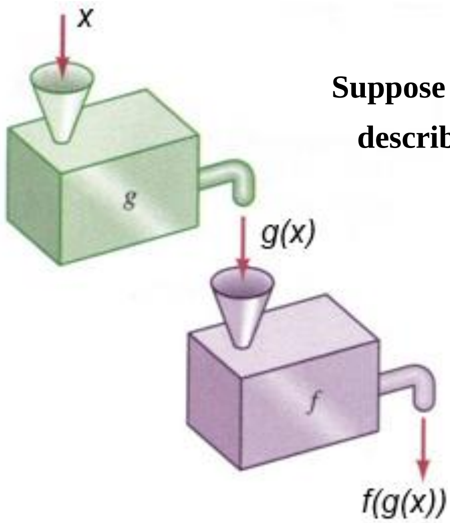
Pre-AP Algebra II
Notes Day #[redacted] 85
Compositions of Functions

Composition of functions: A function operation that uses the output of one function as the <u>input</u> for a second function. Stated another way, a composition of functions uses one function as the input for a different function.



Suppose f and g are functions. Then the composite functions, $f \circ g$, can be described by the equation $(f \circ g)(x) = f(g(x))$.

open dot is a composition of functions, <u>not</u> multiplication

Directions: **For examples 1-4, let** $f(x) = -x^2 + 4$ **and** $g(x) = 4x + 1$ **.**

Ex. 1: $(f \circ g)(-4) = f(g(-4))$

$g(x) = 4x + 1$
$g(-4) = 4(-4) + 1$
$g(-4) = -16 + 1$
$g(-4) = -15$

$f(x) = -x^2 + 4$
$f(g(-4)) = -(-15)^2 + 4$
$f(g(-4)) = -225 + 4$
$\boxed{f(g(-4)) = -221}$

Ex. 2: $(g \circ f)(-4) = g(f(-4))$

$f(x) = -x^2 + 4$
$f(-4) = -(-4)^2 + 4$
$f(-4) = -16 + 4$
$\boxed{f(-4) = -12}$

$g(x) = 4x + 1$
$g(f(-4)) = 4(-12) + 1$
$g(f(-4)) = -48 + 1$
$\boxed{g(f(-4)) = -47}$

Ex. 3: $(f \circ f)(2) = f(f(2))$

$f(x) = -x^2 + 4$
$f(2) = -(2)^2 + 4$
$f(2) = -4 + 4$
$f(2) = 0$

$f(x) = -x^2 + 4$
$f(f(2)) = -(0)^2 + 4$
$\boxed{f(f(2)) = 4}$

Ex. 4: $(f \circ g)(2c) = f(g(2c))$

$g(x) = 4x + 1$
$g(2c) = 4(2c) + 1$
$g(2c) = 8c + 1$

$f(x) = -x^2 + 4$
$f(g(2c)) = -(8c+1)^2 + 4$
$f(g(2c)) = -(8c+1)(8c+1) + 4$
$f(g(2c)) = -(64c^2 + 16c + 1) + 4$
$f(g(2c)) = -64c^2 - 16c - 1 + 4$
$\boxed{f(g(2c)) = -64c^2 - 16c + 3}$

Directions: For examples 5 and 6, let $f(x) = x + 2$ and $g(x) = x^2 + 1$.

Ex. 5: $f(g(x))$

$f(x) = x + 2$

$f(g(x)) = (x^2 + 1) + 2$

$\boxed{f(g(x)) = x^2 + 3}$

Ex. 6: $g(f(x))$

$g(x) = x^2 + 1$

$g(f(x)) = (x + 2)^2 + 1$

$g(f(x)) = (x + 2)(x + 2) + 1$

$g(f(x)) = x^2 + 2x + 2x + 4 + 1$

$\boxed{g(f(x)) = x^2 + 4x + 5}$

Is $f(g(x))$ the same as $g(f(x))$? No

Directions: For example 7, find the composition of $f(x) = 4 - 2x$ with its inverse.

Ex. 7:

A. **Find the inverse of $f(x) = 4 - 2x$**

$y = 4 - 2x$

$x = 4 - 2y$

$-4 \quad -4$

$x - 4 = -2y$

$\frac{x-4}{-2} = \frac{-2y}{-2}$

$-\frac{1}{2}x + 2 = y$

$\boxed{f^{-1}(x) = -\frac{1}{2}x + 2}$

B. **Find $f(f^{-1}(x))$**

$f(x) = 4 - 2x$

$f(f^{-1}(x)) = 4 - 2(-\frac{1}{2}x + 2)$

$f(f^{-1}(x)) = \cancel{4} + x - \cancel{4}$

$\boxed{f(f^{-1}(x)) = x}$

C. **Find $f^{-1}(f(x))$**

$f^{-1}(x) = -\frac{1}{2}x + 2$

$f^{-1}(f(x)) = -\frac{1}{2}(4 - 2x) + 2$

$f(f^{-1}(x)) = -\cancel{2} + x + \cancel{2}$

$\boxed{f(f^{-1}(x)) = x}$

When you take the composition of a function and its inverse, you get x.

Are both these relations functions? yes

<u>Three ways to prove that functions are inverses</u>:

1. **The values of the domain and range are switched.**
2. **The functions are reflections about the line $y = x$.**
3. **The composition of functions yields x.**