Pre-AP Algebra II
Notes Day #[redacted] 84
Inverse Functions and Relations

The inverse of a relation can be found by switching the *x* and *y* coordinates of the ordered pairs. A and B are inverse relations.

$$A=\{(1,\ 5),\ (2,\ 6),\ (3,\ 7)\} \longrightarrow B=\{(5,\ 1),\ (6,\ 2),\ (7,\ 3)\}$$

x y x y x y → y x y x y x

If the relation is written as an equation, you can
<u>find the inverse by literally switching the *x* and *y* and solving for *y*.</u>
The domain of a relation is the range of its inverse relation.
The range of a relation is the domain of its inverse relation.

We use the notation, $f^{-1}(x)$, to denote the inverse of function $f(x)$.   **The "-1" is not an exponent.**

The inverse of a function can be found by exchanging the domain and range as well.

Directions: **Find the inverse of the following equations.**

Ex. 1: $y=3x+1$

$x=3y+1$
$-1 \quad -1$
$x-1=3y$
$\div 3$
$\boxed{\frac{x-1}{3}=y}$

Ex. 2: $y=3-7x$

$x=3-7y$
$-3 \quad -3$
$x-3=-7y$
$\div(-7)$
$\boxed{-\frac{1}{7}x+\frac{3}{7}=y}$

Ex. 3: $g(x)=\frac{x}{2}+3$

$y=\frac{x}{2}+3$
$x=\frac{y}{2}+3$
$-3 \quad -3$
$x-3=\frac{y}{2}$
$\left[x-3=\frac{y}{2}\right]2$
$2x-6=y$
$\boxed{f^{-1}(x)=2x-6}$

Ex. 4: $f(x)=\frac{3x-5}{2}$

$y=\frac{3x-5}{2}$
$x=\frac{3y-5}{2}$
$2\left[x=\frac{3y-5}{2}\right]$
$2x=3y-5$
$+5 \quad +5$
$2x+5=3y$
$\div 3$
$\frac{2}{3}x+\frac{5}{3}=y$
$\boxed{f^{-1}(x)=\frac{2}{3}x+\frac{5}{3}}$

Ex. 5: $f(x)=x^2+9$

$y=x^2+9$
$x=y^2+9$
$-9 \quad -9$
$x-9=y^2$
$\pm\sqrt{x-9}=\cancel{\sqrt{y^2}}$
$\pm\sqrt{x-9}=y$
$\boxed{f^{-1}(x)=\pm\sqrt{x-9}}$
↳ inverse is not a function

Ex. 6: $f(x)=x^2+1$

$y=x^2+1$
$x=y^2+1$
$-1 \quad -1$
$x-1=y^2$
$\pm\sqrt{x-1}=\cancel{\sqrt{y^2}}$
$\pm\sqrt{x-1}=y$
$\boxed{f^{-1}(x)=\pm\sqrt{x-1}}$
↳ inverse is not a function

<u>The inverse of a relation is a relation but the inverse of a function is not necessarily a function.</u>
The vertical line test is used on a graph to determine if it is a function.
~~The horizontal line test will determine if the inverse of a function is a function.~~
A function has an inverse function if and only if each horizontal line intersects the graph of the function in at most one point.
When functions are inverses of each other, they reflect about the line $y = x$ and are symmetric with respect to the line $y = x$.

Ex. 7: Find the inverse of $y = -2x + 4$. Then graph the original equation, its inverse, and the $y = x$ line.

$y = -2x + 4$

$x = -2y + 4$

$-4 \qquad -4$

$x - 4 = -2y$

$\div -2$

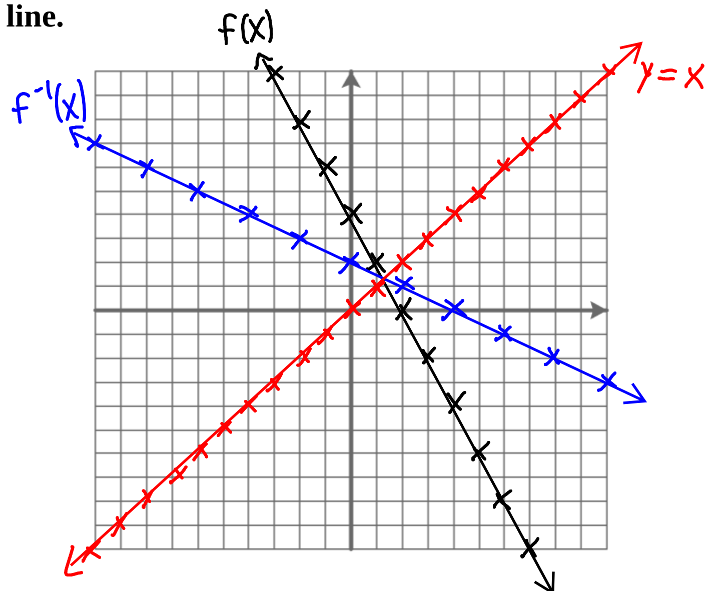
$-\frac{1}{2}x + 2 = y$

$\boxed{f^{-1}(x) = -\frac{1}{2}x + 2}$

Algebra

$f(x)$	$\rightarrow$	$f^{-1}(x)$
(0,4)		(4,0)
(1,2)		(2,1)
(2,0)		(0,2)

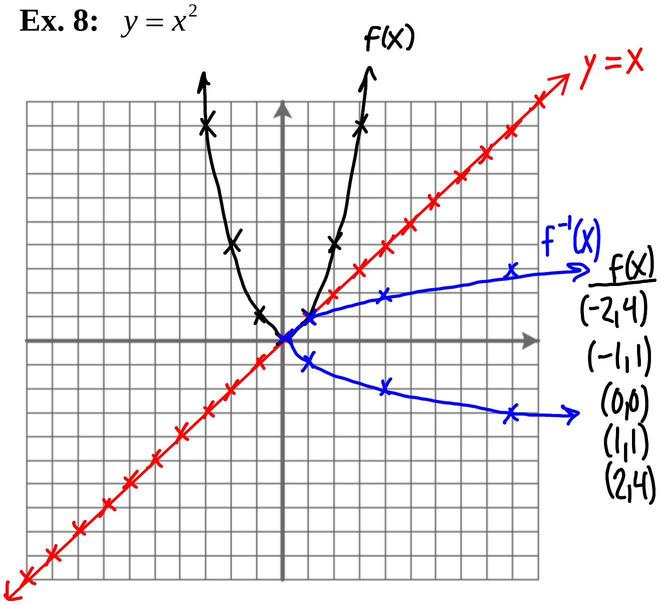
Graphing



Is the original equation a function? yes
Vertical line test
Is the inverse equation a function? yes
Horizontal line test

Directions: Graph the function and then graph its inverse. Also graph the line $y = x$

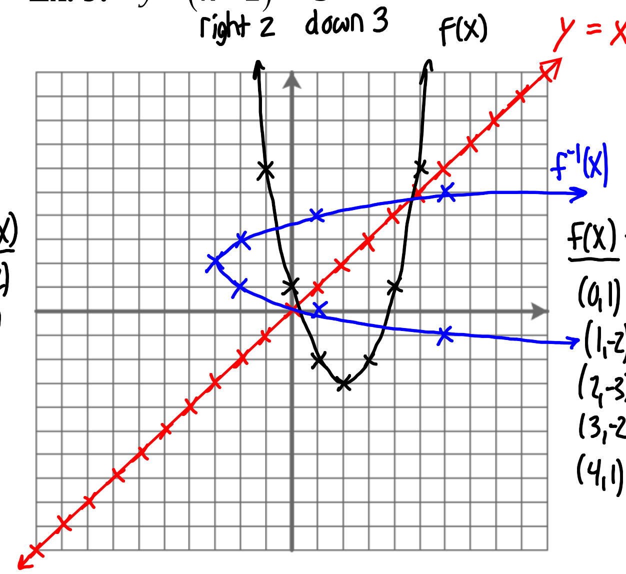
Ex. 8: $y = x^2$



$f(x)$	$\rightarrow$	$f^{-1}(x)$
(-2,4)		(4,-2)
(-1,1)		(1,-1)
(0,0)		(0,0)
(1,1)		(1,1)
(2,4)		(4,2)

Ex. 9: $y = (x-2)^2 - 3$

right 2 down 3



$f(x)$	$\rightarrow$	$f^{-1}(x)$
(0,1)		(1,0)
(1,-2)		(-2,1)
(2,-3)		(-3,2)
(3,-2)		(-2,3)
(4,1)		(1,4)

Is the original equation a function? yes **Is the inverse equation a function?** no