Pre-AP Algebra II
Notes Day #[redacted] 52
Solving Systems of Equations Using Inverse Matrices

<u>We can use inverse matrices to solve equations containing matrices</u>

Directions: **Solve for matrix X in the following equation.**

Ex. 1:
$$\overset{A}{\begin{bmatrix}-2 & -5\\ 1 & 3\end{bmatrix}}X=\overset{B}{\begin{bmatrix}-2\\ 2\end{bmatrix}}$$

First find the inverse of the matrix to the left of matrix X. Then multiply both of the matrices in the equation by the inverse. When you multiply by the inverse, it must be on the <u>LEFT SIDE</u> of each matrix.

$$\overset{A}{\begin{bmatrix}-2 & -5\\ 1 & 3\end{bmatrix}}X=\overset{B}{\begin{bmatrix}-2\\ 2\end{bmatrix}}$$

$$\overset{A^{-1}}{\begin{bmatrix}-3 & -5\\ 1 & 2\end{bmatrix}}\overset{A}{\begin{bmatrix}-2 & -5\\ 1 & 3\end{bmatrix}}X=\overset{A^{-1}}{\begin{bmatrix}-3 & -5\\ 1 & 2\end{bmatrix}}\overset{B}{\begin{bmatrix}-2\\ 2\end{bmatrix}}$$

When you multiply the two matrices on the left side of the equation, you get the <u>identity matrix</u>.
When you multiply the two matrices on the right side of the equation, you get the <u>solution matrix</u>.

Identity Matrix $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}X=\begin{bmatrix}-4\\ 2\end{bmatrix}$ Solution Matrix

When you multiply the identity matrix and matrix X, you get matrix X.

$$X=\begin{bmatrix}-4\\ 2\end{bmatrix}$$

Directions: **Solve for matrix X in the following equation.**

Ex. 2:
$$\overset{A}{\begin{bmatrix}-2 & 3\\ -4 & 5\end{bmatrix}}X=\overset{B}{\begin{bmatrix}6\\ 8\end{bmatrix}}$$

$$\overset{A^{-1}}{\begin{bmatrix}\frac{5}{2} & -\frac{3}{2}\\ 2 & -1\end{bmatrix}}\overset{A}{\begin{bmatrix}-2 & 3\\ -4 & 5\end{bmatrix}}X=\overset{A^{-1}}{\begin{bmatrix}\frac{5}{2} & -\frac{3}{2}\\ 2 & -1\end{bmatrix}}\overset{B}{\begin{bmatrix}6\\ 8\end{bmatrix}}$$

Identity Matrix $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}X=\begin{bmatrix}3\\ 4\end{bmatrix}$ Solution Matrix

$$X=\begin{bmatrix}3\\ 4\end{bmatrix}$$

Directions: **Solve for matrix X in the following equation.**

Ex. 3:

$$\overset{A}{\begin{bmatrix} 0 & 0 & 2 \\ 1 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix}} X = \overset{B}{\begin{bmatrix} 6 \\ -11 \\ 8 \end{bmatrix}}$$

$$\overset{A^{-1}}{\begin{bmatrix} \frac{1}{10} & \frac{2}{5} & \frac{3}{5} \\ \frac{3}{10} & \frac{1}{5} & -\frac{1}{5} \\ \frac{1}{2} & 0 & 0 \end{bmatrix}} \overset{A}{\begin{bmatrix} 0 & 0 & 2 \\ 1 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix}} X = \overset{A^{-1}}{\begin{bmatrix} \frac{1}{10} & \frac{2}{5} & \frac{3}{5} \\ \frac{3}{10} & \frac{1}{5} & -\frac{1}{5} \\ \frac{1}{2} & 0 & 0 \end{bmatrix}} \overset{B}{\begin{bmatrix} 6 \\ -11 \\ 8 \end{bmatrix}}$$

Identity Matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ Solution Matrix

$$X = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

<u>We can use inverse matrices to solve a system of equations</u>.

Ex. 4: **Change this system of equations into the coefficient, variable, and constant matrices.**

$$\overset{A}{\left\{\begin{aligned} 2x - 4y + 2z &= 16 \\ -2x + 5y + 2z &= -34 \\ x - 2y + 2z &= 4 \end{aligned}\right.} \quad B$$

Coefficient matrix

$$A = \begin{bmatrix} 2 & -4 & 2 \\ -2 & 5 & 2 \\ 1 & -2 & 2 \end{bmatrix}$$

Variable matrix

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Constant matrix

$$B = \begin{bmatrix} 16 \\ -34 \\ 4 \end{bmatrix}$$

Directions: **Solve the following system of equations using the inverse matrix method.**

Ex. 5: $\begin{cases} 2x + 4y = 10 \\ 3x + 5y = 14 \end{cases}$

$$\begin{matrix} \mathbf{A} & \mathbf{X} & = & \mathbf{B} \\ \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} & \begin{bmatrix} x \\ y \end{bmatrix} & = & \begin{bmatrix} 10 \\ 14 \end{bmatrix} \end{matrix}$$

$$\begin{matrix} \mathbf{A}^{-1} & \mathbf{A} & \mathbf{X} & = & \mathbf{A}^{-1} & \mathbf{B} \\ \begin{bmatrix} -\frac{5}{2} & 2 \\ \frac{3}{2} & -1 \end{bmatrix} & \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} & \begin{bmatrix} x \\ y \end{bmatrix} & = & \begin{bmatrix} -\frac{5}{2} & 2 \\ \frac{3}{2} & -1 \end{bmatrix} & \begin{bmatrix} 10 \\ 14 \end{bmatrix} \end{matrix}$$

Identity Matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ Solution Matrix

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$(\underset{x}{\underline{\ 3\ }}, \underset{y}{\underline{\ 1\ }})$

Directions: **Solve the following system of equations using the inverse matrix method.**

Ex 6:
$$\begin{cases} 2x+3y+z=-1 \\ 3x+3y+z=\ \ 1 \\ 2x+4y+z=-2 \end{cases}$$

$$\underset{\mathbf{A}}{\begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}} \underset{\mathbf{X}}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{\mathbf{B}}{\begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}}$$

$$\underset{\mathbf{A^{-1}}}{\begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 6 & -2 & -3 \end{bmatrix}} \underset{\mathbf{A}}{\begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}} \underset{\mathbf{X}}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{\mathbf{A^{-1}}}{\begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 6 & -2 & -3 \end{bmatrix}} \underset{\mathbf{B}}{\begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}}$$

Identity Matrix
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$$
Solution Matrix

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$$

(2 , -1 , -2)

x , y , z