Pre-AP Algebra II

Notes Day #[redacted] 50-51

Matrix Basics

A matrix **is a rectangular array of numbers enclosed by brackets.**

$$\begin{bmatrix} 2 & 5 \\ 1 & 0 \end{bmatrix}$$

The numbers in a matrix are called the <u>elements</u> of the matrix.

$\begin{bmatrix} 2 & 5 \\ 1 & 0 \end{bmatrix}$ **2, 5, 1, and 0 are the elements of this matrix**

The number of <u>rows</u> (horizontal) and the number of <u>columns</u> (vertical) determine the dimensions **of the matrix.**

When giving the dimensions of a matrix, you <u>always</u> write the number of rows first, then the columns (<u>rows X columns</u>).

$$\begin{array}{cc} & \text{3 columns} \\ \text{2 Rows} & \begin{bmatrix} 5 & 7 & -9 \\ 9 & 5 & -3 \end{bmatrix} \end{array}$$

This matrix is a 2 row by 3 column matrix, a 2 X 3

In a <u>square matrix</u>, the number of rows and the number of columns are equal.

$\begin{bmatrix} 4 & 6 \\ -1 & 7 \end{bmatrix}$ **a 2 X 2 matrix**

<u>Capital letters</u> are used to name matrices.

$$A = \begin{bmatrix} 1 & 9 & 3 \\ 4 & 7 & 8 \end{bmatrix}$$

If all the elements of a matrix are zeros, the matrix is called a zero matrix**.**

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

<u>Two matrices are equal if and only if they have the same dimensions and the elements in all corresponding positions are equal.</u>

$$\begin{bmatrix} 2 & 3 \\ 8 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 8 & 1 \end{bmatrix}$$

Matrices must be the same size to add or subtract them.
Simply add or subtract the corresponding elements.

Ex. 1: $\begin{bmatrix} 1 & -2 & 0 \\ 3 & -5 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 9 & -3 \\ -9 & 6 & 12 \end{bmatrix} = \begin{bmatrix} 4 & 7 & -3 \\ -6 & 1 & 19 \end{bmatrix}$

Ex. 2: $\begin{bmatrix} 3 & 2 & 4 \\ -1 & 4 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 4 & 3 \\ -2 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & -4 \end{bmatrix}$

Directions: Solve for matrix X.

Ex. 3: $X - \underbrace{\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 0 & 1 \\ 8 & 9 \end{bmatrix}}_{B}$

$+\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} \quad +\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

$\boxed{X = \begin{bmatrix} 1 & 2 \\ 11 & 11 \end{bmatrix}}$

$X - A = B$
$+A \quad +A$
$X = B + A$

Directions: Solve for x and y.

Ex. 4: $\begin{bmatrix} 2x-5 & 4 \\ 3 & 3y+12 \end{bmatrix} = \begin{bmatrix} 25 & 4 \\ 3 & y+18 \end{bmatrix}$

$2x - 5 = 25$
$+5 \quad +5$
$\frac{2x}{2} = \frac{30}{2}$
$\boxed{x = 15}$

$3y + 12 = y + 18$
$-y \quad -y$
$2y + 12 = 18$
$-12 \quad -12$
$\frac{2y}{2} = \frac{6}{2}$
$\boxed{y = 3}$

Ex. 5: $\begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} x & 5 \\ 0 & y \end{bmatrix}$

$\begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} x & 5 \\ 0 & y \end{bmatrix}$ $\boxed{\begin{matrix} x=1 \\ y=2 \end{matrix}}$

You can also multiply a matrix by a real number

Ex. 6: $3\begin{bmatrix} 3 & 5 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 6 & 24 \end{bmatrix}$

3 is called a scalar.

Directions: Solve for matrix X.

Ex. 7: $4X + 2\begin{bmatrix} 3 & 4 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 4 & 2 \end{bmatrix}$

$4X + \begin{bmatrix} 6 & 8 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 4 & 2 \end{bmatrix}$
$-\begin{bmatrix} 6 & 8 \\ -4 & 2 \end{bmatrix} \quad -\begin{bmatrix} 6 & 8 \\ -4 & 2 \end{bmatrix}$

$\frac{1}{4}\left[4X = \begin{bmatrix} 4 & -8 \\ 8 & 0 \end{bmatrix} \right]$

$\boxed{X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}}$

<u>To multiply matrices, the number of columns in the first matrix has to equal the number of rows in the second matrix.</u>

Ex. 8: $\overset{A}{\begin{bmatrix} -3 & 3 \\ 5 & 0 \end{bmatrix}}\overset{B}{\begin{bmatrix} -1 & 0 \\ 3 & -4 \end{bmatrix}} = \begin{bmatrix} 12 & -12 \\ -5 & 0 \end{bmatrix}$

Ex. 9: $\overset{B}{\begin{bmatrix} -1 & 0 \\ 3 & -4 \end{bmatrix}}\overset{A}{\begin{bmatrix} -3 & 3 \\ 5 & 0 \end{bmatrix}} = \begin{bmatrix} 3 & -3 \\ -29 & 9 \end{bmatrix}$

The two matrices in examples 8 and 9 are the same but in a different order.
Notice we have different products for the multiplication.
<u>Matrix multiplication is not commutative.</u>
The order in which matrices are multiplied matters.

Ex. 10: $\begin{bmatrix} 2 & -1 & 4 \\ 3 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 2 & 4 \\ -1 & 6 \end{bmatrix} = \begin{bmatrix} -4 & 16 \\ 1 & 6 \end{bmatrix}$

Ex. 11: $\begin{bmatrix} 0 & 2 & -1 \\ 4 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 3 & 0 \\ -1 & 0 & 2 \\ 1 & 0 & -2 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 6 \\ 15 & 12 & 2 \\ 3 & 0 & -6 \end{bmatrix}$

<u>MATRIX DETERMINANTS AND INVERSES</u>

The Determinant

The determinant of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ *is* $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Directions: **Find the value of the following determinant.**

Ex. 12: $\begin{vmatrix} -3 & 4 \\ 2 & -5 \end{vmatrix} = \quad 15 - 8 \quad = \boxed{7}$

Ex. 13: $\begin{vmatrix} 2 & 4 & 1 \\ 1 & 6 & 3 \\ -2 & 3 & 5 \end{vmatrix} = \boxed{13}$

If the determinant is equal to 0, then there is no inverse. Determinants help us find the inverses!

Recall, in the real number system, 1 is the multiplication identity since $1 \bullet a = a$ **.**

The multiplicative inverse of a is $\frac{1}{a}$ **since** $\frac{1}{a} \bullet a = 1$**, the identity.**

For matrices, the multiplicative identity is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **for a 2x2 matrix since**

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The identity matrix for a 3x3 is: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

<u>If you multiply a matrix and the inverse of that matrix, you will have the identify matrix.</u>

To show that two matrices are inverses of each other, we must show that

$$A^{-1} \bullet A = I$$

Directions: **Determine if the following matrices are inverses. <u>If you do not get the identity matrix, then they are not inverses.</u>**

Ex. 14: $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\boxed{\text{Inverses}}$

Ex. 15: $\begin{bmatrix} 7 & 2 & -9 \\ 3 & 1 & -4 \\ -2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -4 & 2 \\ -2 & 5 & 2 \\ 1 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 6 & -2 \end{bmatrix}$ $\boxed{\text{Not inverses}}$

The inverse of the 2x2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ **is:** $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Directions: **Find the inverse of each of the following matrices.**

Ex. 16: $\begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}$

Ex. 17: $\begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$ $\boxed{\text{No inverse}}$

Ex. 18: $\begin{bmatrix} 12 & -9 & 13 \\ 0 & 0 & 8 \\ -9 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{57} & \frac{35}{456} & -\frac{3}{19} \\ -\frac{3}{19} & \frac{43}{152} & -\frac{4}{19} \\ 0 & \frac{1}{8} & 0 \end{bmatrix}$