Pre-AP Algebra II
Notes Day #[redacted] 45-46
Solving Linear Systems of Equations
Elimination & Substitution Method

<u>Elimination Method</u>

In a system of equations where the *<u>coefficients</u>* of the *x* or *y* terms are the *<u>same</u>* number but have *<u>different</u> <u>signs</u>*, you can solve the system by adding the two equations. Because one of the variables is eliminated, this method is called *<u>elimination</u>*.

Ex. 1: A $\begin{cases} x-3y=7 \\ 3x+3y=9 \end{cases}$ B

A+B: $4x = 16$

$\frac{4x}{4} = \frac{16}{4}$

$\boxed{x=4}$

A $x-3y=7$

$4-3y=7$

$-4 \quad -4$

$-3y = 3$

$\frac{-3y}{-3} = \frac{3}{-3}$

$\boxed{y=-1}$

Solution must be written as an ordered pair

$\boxed{(4,-1)}$

If the two *<u>coefficients</u>* of the x or y terms are the *<u>same</u>* and have the *<u>same</u> <u>sign</u>*, then multiply one of the equations by (-1) so the signs will be *<u>different</u>* and then *<u>add</u>* the two equations together. This is the elimination method using multiplication and addition.

Ex. 2: A $\begin{cases} 6x+5y=4 \\ (6x-7y=-20)\,{-1} \end{cases}$ B

B $-6x+7y=20$

A+B: $12y = 24$

$\frac{12y}{12} = \frac{24}{12}$

$\boxed{y=2}$

A $6x+5y=4$

$6x+5(2)=4$

$6x+10=4$

$-10 \quad -10$

$6x=-6$

$\frac{6x}{6} = \frac{-6}{6}$

$\boxed{x=-1}$

$\boxed{(-1,2)}$

Some systems of equations *<u>cannot</u>* be solved simply by adding the equations or by multiplying by a (-1) and then adding the equations. Sometimes *<u>one</u>* or *<u>both</u>* equations must first be *<u>multiplied by a number</u>* before the system can be solved by elimination.

Ex. 3: A $\begin{cases} 2x+3y=6 \\ (x+2y=5)\,{-2} \end{cases}$ B

B $-2x-4y=-10$

A+B: $-y = -4$

$\frac{-y}{-1} = \frac{-4}{-1}$

$\boxed{y=4}$

B $x+2y=5$

$x+2(4)=5$

$x+8=5$

$-8 \quad -8$

$\boxed{x=-3}$

$\boxed{(-3,4)}$

Directions: **Solve using the elimination method.**

Ex. 4: $\begin{cases} A\ (5x-2y=3)\ 7 \\ B\ (2x+7y=9)\ 2 \end{cases}$

A $35x - 14y = 21$
B $4x + 14y = 18$

A+B $\dfrac{39x}{39} = \dfrac{39}{39}$

$\boxed{x = 1}$

B $2x + 7y = 9$
$2(1) + 7y = 9$
$2 + 7y = 9$
$-2 \qquad -2$
$\dfrac{7y}{7} = \dfrac{7}{7}$
$\boxed{y = 1}$

$\boxed{(1, 1)}$

Substitution Method

To use the substitution method to solve a system of linear equations, follow these steps:

1. **Solve for one of the two variables using one of the equations if neither of the equations is already in this form.**
2. **Substitute this expression into the other equation for the variable you just solved for.**
3. **Solve for the other variable.**
4. **After you find the value for this variable, substitute this value in the equation in Step 1.**
5. **Solve for the second variable.**
6. **Check the values in both equations.**

Directions: **Use the substitution method to solve for the following systems of equations.**

Ex. 5: $\begin{cases} A\ y = 2x - 5 \\ B\ 3x + y = 10 \end{cases}$

A → B $3x + y = 10$
$3x + (2x-5) = 10$
Note: Only 1 variable after substitution
$5x - 5 = 10$
$+5 \quad +5$
$\dfrac{5x}{5} = \dfrac{15}{5}$
B $\boxed{x = 3}$
Use A to find y

A $y = 2x - 5$
$y = 2(3) - 5$
$y = 6 - 5$
$\boxed{y = 1}$

$\boxed{(3, 1)}$

Ex. 6: $\begin{cases} A\ 3x + y = 4 \\ B\ -6x - 2y = -8 \end{cases}$

A $3x + y = 4$
$-3x \qquad -3x$
A $y = -3x + 4$

A → B $-6x - 2y = -8$
$-6x - 2(-3x+4) = -8$
$-\cancel{6x} + \cancel{6x} - 8 = -8$
$-8 = -8$
True,
$\boxed{\text{Infinitely many solutions}}$

Directions: **Solve using the substitution method.**

Ex. 7:
$$\begin{cases} 3x+2y=1 & (A) \\ x-y=2 & (B) \end{cases}$$

$+y \quad +y$

(B) $x = y+2$

(B) → (A) $3x + 2y = 1$

$3(y+2) + 2y = 1$

$3y + 6 + 2y = 1$

$5y + 6 = 1$

$-6 \quad -6$

$\frac{5y}{5} = \frac{-5}{5}$

(A) $\boxed{y = -1}$

(B) $x - y = 2$

$x - (-1) = 2$

$x + 1 = 2$

$-1 \quad -1$

$\boxed{x = 1}$

$\boxed{(1, -1)}$

Ex. 8:
$$\begin{cases} 2x+2y=6 & (A) \\ 5x=-5y+10 & (B) \end{cases}$$

$\div 5$

(B) $x = -y + 2$

(B) → (A) $2x + 2y = 6$

$2(-y+2) + 2y = 6$

$-\cancel{2y} + 4 + \cancel{2y} = 6$

$4 = 6$

False,

$\boxed{\text{No Solution}}$

Ex. 9: **An exam worth 145 points contains 50 questions. Some of the questions are worth two points and some are worth five points. How many two point questions are on the test? How many five point questions are on the test? <u>Solve by elimination</u>.**

Let x = # two point questions

y = # five point questions

(A) $(x + y = 50) \cdot -2$

(B) $2x + 5y = 145$

(A) $-2x - 2y = -100$

(B) + (A) $\frac{3y}{3} = \frac{45}{3}$

$\boxed{y = 15}$

(A) $x + y = 50$

$x + 15 = 50$

$-15 \quad -15$

$\boxed{x = 35}$

$\boxed{(35, 15)}$

Ex. 10: **The Lakers scored a total of 80 points in a basketball game against the Bulls. The Lakers made a total of 37 two-point and three-point baskets. How many two-point shots did the Lakers make? How many three-point shots did the Lakers make? <u>Solve by elimination</u>.**

x = # two-point baskets made

y = # three-point baskets made

A $(x + y = 37)\ -2$

B $2x + 3y = 80$

A $-2x - 2y = -74$

B + A $\boxed{y = 6}$

A $x + y = 37$

$x + 6 = 37$

$-6 \quad -6$

$\boxed{x = 31}$

$\boxed{(31, 6)}$