Pre-AP Algebra II
Notes Day #32-33
Parent Functions and Rigid Transformations

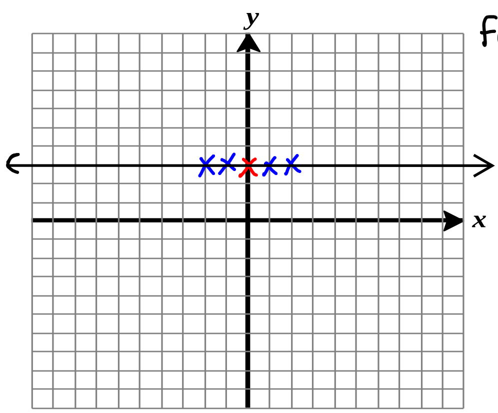
A family of graphs is a group of graphs that display one or more similar characteristics. The __parent function__ is the simplest of the graphs in a family of functions. It is the graph of the function with no __transformations__.

Parent Functions

Directions: **Graph each of the following parent functions and state the domain and range.**

Ex. 1: Constant function $f(x) = a$ **[Pick a #]**

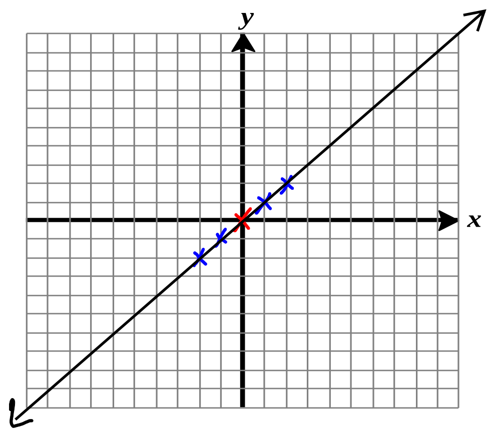
$f(x) = 3$



Domain: $\{x \mid x \in \mathbb{R}\}$, $(-\infty, \infty)$

Range: $\{y \mid y = 3\}$, $[3]$

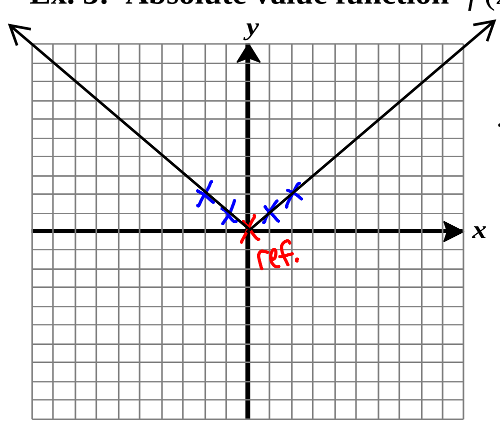
Ex. 2: Identity or linear function $f(x) = x$



Domain: $\{x \mid x \in \mathbb{R}\}$, $(-\infty, \infty)$

Range: $\{y \mid y \in \mathbb{R}\}$, $(-\infty, \infty)$

Ex. 3: Absolute value function $f(x) = |x|$

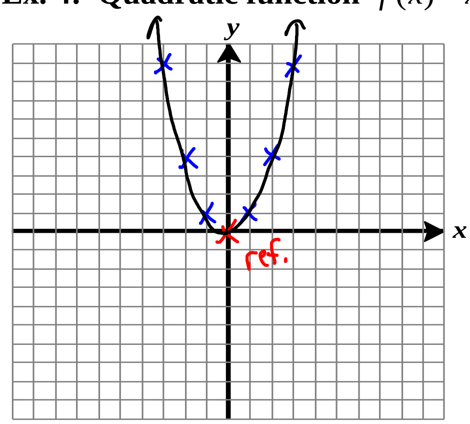


x	y
-2	2
-1	1
0	0
1	1
2	2

Domain: $\{x \mid x \in \mathbb{R}\}$, $(-\infty, \infty)$

Range: $\{y \mid y \geq 0\}$, $[0, \infty)$

Ex. 4: Quadratic function $f(x) = x^2$



x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

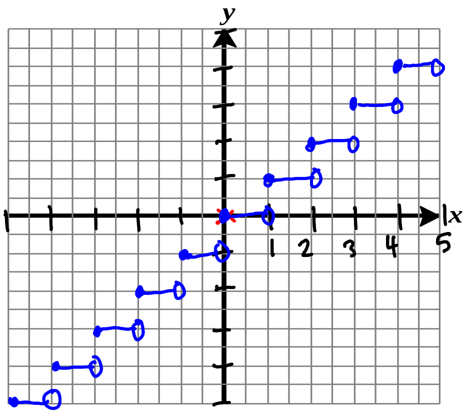
Domain: $\{x \mid x \in \mathbb{R}\}$, $(-\infty, \infty)$

Range: $\{y \mid y \geq 0\}$, $[0, \infty)$

Directions: **Graph the following parent function and state the domain and range.**

The greatest integer function is a type of step function (which is a piecewise function). The symbol $\lfloor x \rfloor$ means the greatest integer less than or equal to *x*. For example, $\lfloor 3.25 \rfloor = 3$, $\lfloor -4.6 \rfloor = -5$, and $\lfloor 4 \rfloor = 4$. In short it means always round down. Hence, it is also called a flooring function.

Ex. 5: Greatest Integer Function $f(x) = \lfloor x \rfloor$



Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \in \mathbb{Z}\}$

A piecewise **function is defined not by a single equation, but by two or more. Each equation is valid for some** interval **.**

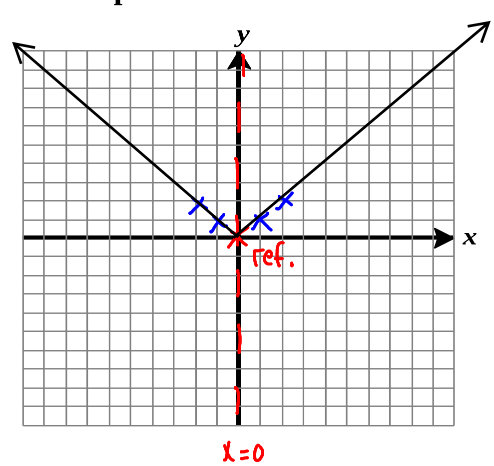
Earlier we discussed functions being "a rule" for how to map the domain onto the range. Piecewise-defined functions may be thought of as having more rules than the one rule functions you are used to working with.

You have already seen an example of a piecewise-defined function. Consider the parent function for absolute value**,** $f(x) = |x|$**. A proper definition for absolute value appears in Example 6.**

Ex. 6: Graph the absolute value function by creating a table from the piecewise definition.

$$|x| = \begin{cases} -x \text{ if } x < 0 \\ x, \text{ if } x \geq 0 \end{cases}$$

x	*f(x)*	Rule?
-2	2	-x
-1	1	-x
0	0	x
1	1	x
2	2	x



We will discuss piecewise functions in more detail at a later date. It is an important point to introduce here since piecewise functions typically begin with absolute value which we have just learned how to graph.

Transformations of Parent Functions

Transformations of a parent graph may either be rigid **or** non-rigid**. Rigid transformations preserve a graph's** shape **but change its location in the coordinate plane. Another way to state that the graph's shape is preserved is that the distance between any two sets of points on the graph is the same before and after the transformation.**

Specifically, the location of the reference point from which the pattern is graphed changes in most rigid transformations. A reference point always starts at $(0,0)$ for every function and changes with translations**. The reference point may or may not be a part of the function's actual graph. We will focus on rigid transformations first because they are simpler.**

Standard Form Transformation Equations

Absolute value function:	$f(x) = a\lvert b(x-c)\rvert + d$
Quadratic function:	$f(x) = a\left[b(x-c)\right]^2 + d$
Greatest integer function:	$f(x) = a\lfloor b(x-c) \rfloor + d$

There are more types of functions that we will study at a later date but we will begin with these three for practicing transformations.

All of the parent functions are transformed in the same manner. All of the parameters (a, b, c, d) have the same effects on the graph of $f(x)$ in every case. The only difference between graphs are the shapes **of their parent function pattern.**

The parent pattern is derived from evaluating the function over a set of values which you can do if you forget what the parent pattern of a function is.

a **and** d **effect the *y*-values of the function. Looking at the above functions you can see this as $a\ f(x) + d$. These transformations occur on the outside of the function so it effects the** outputs **of the function.**

b **and** c **effect the *x*-values of the function. Looking at the above functions you can see this as $f(b(x-c))$. These transformations occur on the inside of the function so it effects the** inputs **of the function. Important to note is that the changes for the inputs have the** opposite **effect from what you might expect. This means adding goes left (because of the subtraction sign in the formula), and multiplying by 2 is actually a compression by $\frac{1}{2}$.**

As stated we will focus on the rigid transformations first which have two different types: Translations and reflections.

Translations

To shift a graph right, left, up, or down

horizontal translation

if $c>0$, the shift is to the right
if $c<0$, the shift is to the left

vertical translation

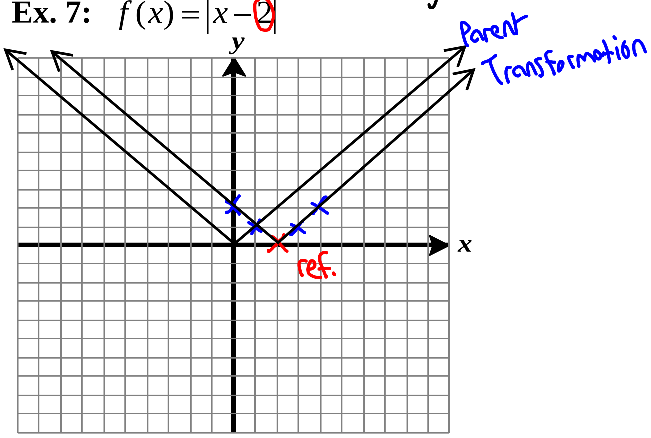
if $d>0$, the shift is upward
if $d<0$, the shift is downward

Note: Be careful of the value for c. Looking at the transformation equation $f(x)=a|b(x-c)|+d$ the subtraction sign means that $f(x)=|x-2|$ has a value of $+2$ for c and not -2.

Directions: Graph each of the following functions and state the domain and range.

c=2 right 2

Ex. 7: $f(x)=|x-2|$

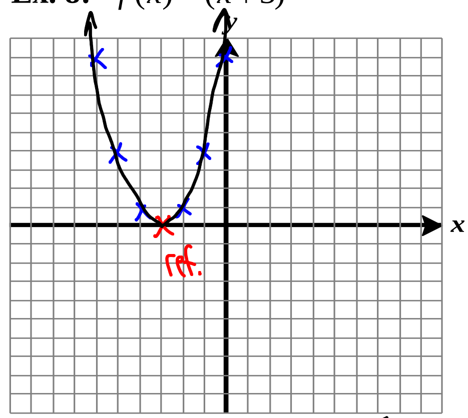


Domain: $(-\infty,\infty)$, $\{x|x\in\mathbb{R}\}$

Range: $[0,\infty)$, $\{y|y\geq 0\}$

left 3

Ex. 8: $f(x)=(x+3)^2$

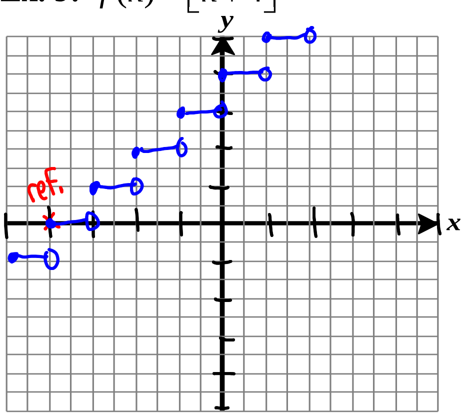


Domain: $(-\infty,\infty)$, $\{x|x\in\mathbb{R}\}$

Range: $[0,\infty)$, $\{y|y\geq 0\}$

left 4

Ex. 9: $f(x)=\lfloor x+4 \rfloor$

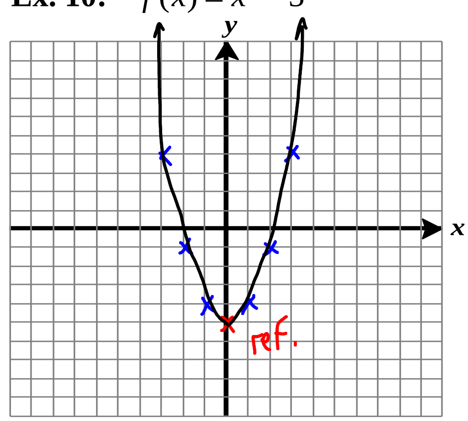


Domain: $\{x|x\in\mathbb{R}\}$

Range: $\{y|y\in\mathbb{Z}\}$

down 5

Ex. 10: $f(x)=x^2-5$

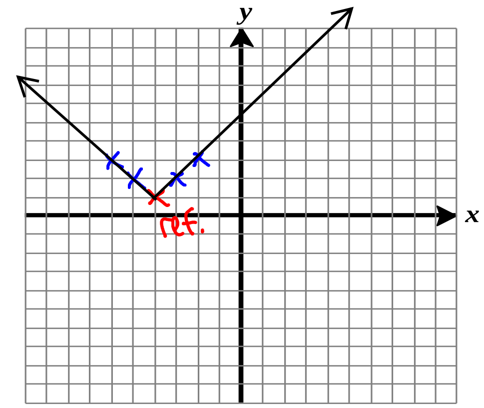


Domain: $(-\infty,\infty)$, $\{x|x\in\mathbb{R}\}$

Range: $[-5,\infty)$, $\{y|y\geq -5\}$

Directions: Graph each of the following functions and state the domain and range.

Ex. 11: $f(x) = |x+4| + 1$ left 4 → up 1



Domain: $(-\infty, \infty)$, $\{x \mid x \in \mathbb{R}\}$

Range: $[1, \infty)$, $\{y \mid y \geq 1\}$

Reflections

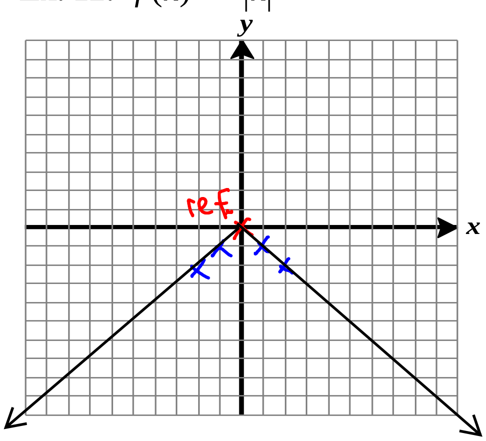
flip over the *x*-axis or *y*-axis

a **flips the graph over the *x*-axis if** $a < 0$ $\qquad$ b **flips the graph over the *y*-axis if** $b < 0$

Important Note: **The coefficient of** x **must be factored out from all terms inside of the function to identify the proper transformations.**

Directions: Graph each of the following functions and state the domain and range.

Ex. 12: $f(x) = -|x|$ reflect over x-axis



Domain: $(-\infty, \infty)$, $\{x \mid x \in \mathbb{R}\}$

Range: $(-\infty, 0]$, $\{y \mid y \leq 0\}$

Parent

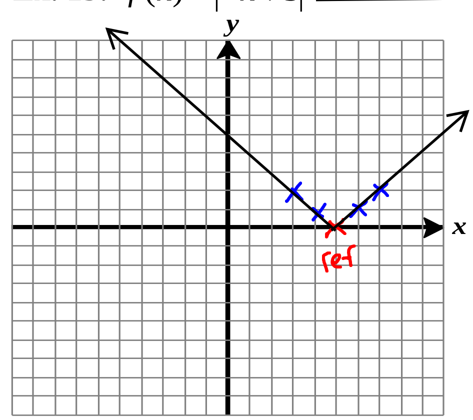
x	y
-2	2
-1	1
0	0
1	1
2	2

-1 → New

x	y
-2	-2
-1	-1
0	0
1	-1
2	-2

Ex. 13: $f(x) = |-x+5|$ $\xrightarrow{\text{factor } -1 \text{ from } x}$ $f(x) = |-1(x-5)|$

reflect over y-axis $\qquad$ right 5



Domain: $(-\infty, \infty)$, $\{x \mid x \in \mathbb{R}\}$

Range: $[0, \infty)$, $\{y \mid y \geq 0\}$

Parent

x	y
-2	2
-1	1
0	0
1	1
2	2

-1 → New

x	y
2	2
1	1
0	0
-1	1
-2	2

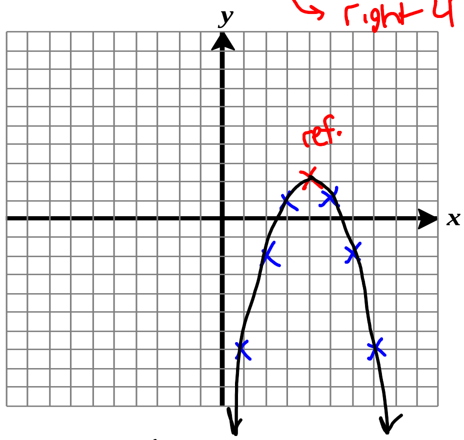
Same table before and after [y-axis symmetry]

Directions: Graph each of the following functions and state the domain and range.

reflect over x-axis

Ex. 14: $f(x) = -(x-4)^2 + 2$ → up 2

right 4

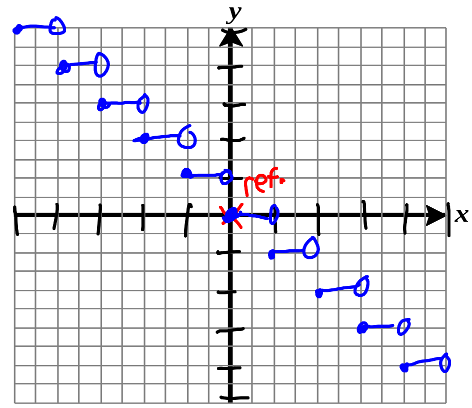


Domain: $(-\infty, \infty)$, $\{x \mid x \in \mathbb{R}\}$

Range: $(-\infty, 0]$, $\{y \mid y \leq 0\}$

reflect over x-axis

Ex. 15: $f(x) = -\lfloor x \rfloor$



Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \in \mathbb{Z}\}$

Parent →

x	y
-2	4
-1	1
0	0
1	1
2	4

-1

New

x	y
2	-4
1	-1
0	0
1	-1
2	-4