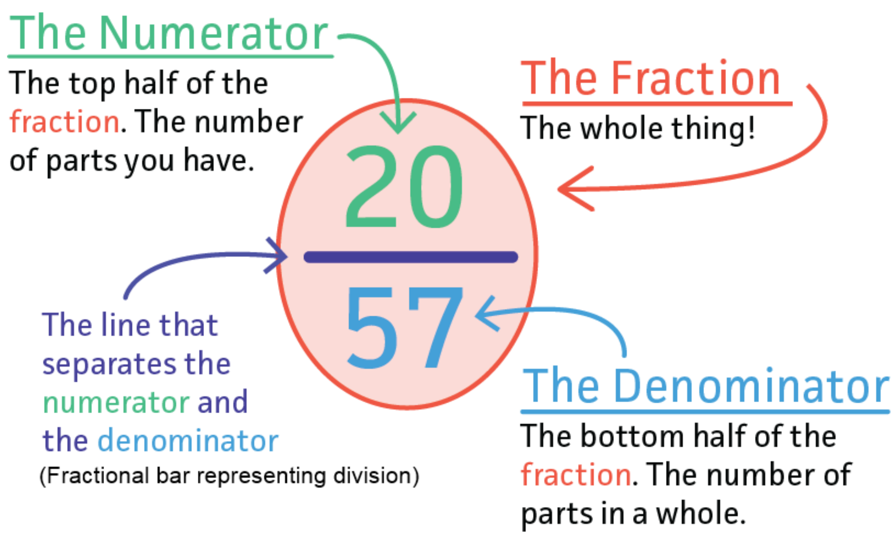
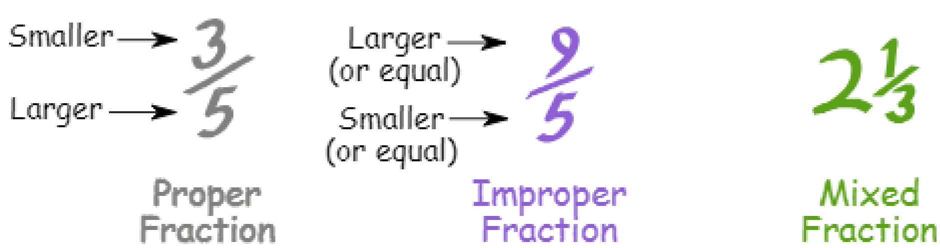
Pre-AP Algebra II
Notes Day #6
Fractions

<u>Fraction</u> - a numerical quantity that is not a whole number and can be written as the ratio of two integers.

<u>Classifying the parts of a fraction</u>:



<u>Types of Fractions</u>:



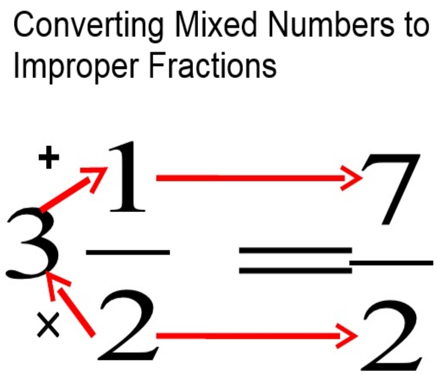
- **<u>Proper fraction</u> - A fraction whose numerator is strictly less than the denominator.**
- **<u>Improper fraction</u> - A fraction whose numerator is greater than or equal to the denominator.**
- **<u>Mixed fraction/Mixed number</u> - A number consisting of an integer representing the whole portions and a proper fraction representing the non-whole parts.**

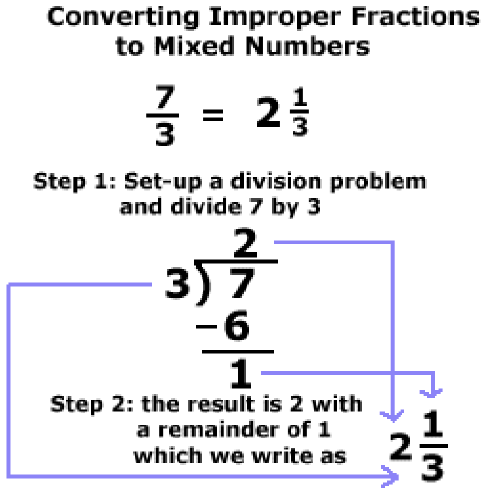
Directions: **Classify the following fractions.**

Ex. 1: $\frac{7}{9}$ Proper **Ex. 2:** $3\frac{1}{4}$ Mixed **Ex. 3:** $\frac{17}{12}$ Improper

Ex. 4: $8\frac{1}{5}$ Mixed **Ex. 5:** $\frac{32}{7}$ Improper **Ex. 6:** $\frac{1}{3}$ Proper

Converting between types of fractions:





Directions: **If the expression is a mixed number, convert it to an improper fraction. If the expression is an improper fraction, convert it to a mixed number.**

Ex. 7: $4\frac{1}{8} = \boxed{\frac{33}{8}}$

Ex. 8: $\frac{15}{4} = \boxed{3\frac{3}{4}}$

Ex. 9: $7\frac{3}{7} = \boxed{\frac{52}{7}}$

Ex. 10: $\frac{13}{6}$ $\boxed{2\frac{1}{6}}$

Reducing fractions:

When the numerator and denominator of a fraction have a factor **in common, it can be cancelled out from both to reduce the fraction. The result is a fraction whose numerator and denominator are both less than what they originally were. Stated abstractly:**

$$\frac{ac}{bc} = \frac{a}{b} \bullet \frac{c}{c} = \frac{a}{b} \bullet 1 = \frac{a}{b}$$ **where** c **is a common factor.**

Example: $$\frac{15}{9} = \frac{5 \bullet \cancel{3}}{3 \bullet \cancel{3}} = \frac{5}{3}$$

Directions: **Reduce the following fractions.**

Ex. 11: $\frac{12}{8} = \frac{3 \cdot \cancel{4}}{2 \cdot \cancel{4}} = \boxed{\frac{3}{2}}$

Ex. 12: $\frac{28}{21} = \frac{4 \cdot \cancel{7}}{3 \cdot \cancel{7}} = \boxed{\frac{4}{3}}$

Ex. 13: $-\frac{72}{56} = -\frac{9 \cdot \cancel{8}}{7 \cdot \cancel{8}} = \boxed{-\frac{9}{7}}$

<u>Operations with fractions</u>:

- **Multiplication:**

We will begin with multiplying fractions because it is the most straight forward of basic arithmetic.

$$\frac{a}{c} \bullet \frac{b}{d} = \frac{ab}{cd}$$

Very simply multiply the numerators together and multiply the denominators together. Whether you reduce before you multiply or after is up to you, though doing so beforehand will lead to dealing with smaller numbers when multiplying.

Directions: **Multiply the following fractions.**

Ex. 14: $\frac{2}{7} \bullet -\frac{3}{5} = \boxed{-\frac{6}{35}}$

Ex. 15: $\frac{x}{2} \bullet \frac{3x}{(x+4)} = \boxed{\frac{3x^2}{2(x+4)} \text{ or } \frac{3x^2}{2x+8}}$

- **Adding and subtracting:**

It is important to remember when adding and subtracting fractions that they must have a <u>common denominator</u>.

ADDING FRACTIONS

FORMULA

With same denominators

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

With different denominators

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

Subtraction works the same way, only the plus signs should be replaced with minus signs in the above formula. <u>A point of note is that *bd* is a common denominator but it may not be the *least* common denominator of both fractions.</u>

Directions: **Add or subtract the following fractions.**

Ex. 16: $\frac{3}{4} + \frac{2}{5} = \left(\frac{5}{5}\right)\frac{3}{4} + \left(\frac{4}{4}\right)\frac{2}{5}$

$= \frac{15}{20} + \frac{8}{20}$

$= \boxed{\frac{23}{20}}$

Ex. 17: $\frac{3}{2} - \frac{17}{15} = \left(\frac{15}{15}\right)\frac{3}{2} - \left(\frac{2}{2}\right)\frac{17}{15}$

$= \frac{45}{30} - \frac{34}{30}$

$= \boxed{\frac{11}{30}}$

Directions: Add or subtract the following fractions.

Ex. 18: $\frac{2x}{5}+\frac{1}{7}=\left(\frac{7}{7}\right)\frac{2x}{5}+\left(\frac{5}{5}\right)\frac{1}{7}$

$=\frac{14x}{35}+\frac{5}{35}$

$=\boxed{\frac{14x+5}{35}}$

Ex. 19: $\frac{7x}{x+5}-\frac{4x}{x+5}=\boxed{\frac{3x}{x+5}}$

- **Division:**

For dividing any two fractions:

Keep-change-flip

KCF KCF

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\bullet\frac{d}{c}\quad or\quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a}{b}\bullet\frac{d}{c}$$

A Complex **fraction is a fraction that contains another fraction in either its numerator, denominator, or both. A fraction divided by another fraction sounds difficult, but the result is simply multiplying the numerator by the reciprocal of the denominator.**

Why is this the case? It actually follows directly from the field axioms of Algebra. By the identity property, multiplying by 1 does not change the value of an expression. We will use this then and rewrite the number one in a clever way to allow us to cancel out the denominator. Specifically we will rewrite the number one as $\frac{d}{c}$ **divided by** $\frac{d}{c}$**, because any expression divided by the same expression is one (a result of the multiplicative inverse axiom). This leads to a simplification and we obtain the below result:**

$$\frac{\frac{a}{b}}{\frac{c}{d}}\bullet 1=\frac{\frac{a}{b}}{\frac{c}{d}}\bullet\frac{\frac{d}{c}}{\frac{d}{c}}=\frac{\frac{a}{b}}{\frac{\cancel{c}}{\cancel{d}}}\bullet\frac{\frac{d}{c}}{\frac{\cancel{d}}{\cancel{c}}}=\frac{a}{b}\bullet\frac{d}{c}$$

Directions: Divide the following fractions.

Ex. 20: KCF $\frac{2}{5}\div\frac{7}{3}=\frac{2}{5}\bullet\frac{3}{7}$

$=\boxed{\frac{6}{35}}$

Ex. 21: K C F $\frac{\frac{3}{4}}{\frac{5}{2}}=\frac{3}{4}\cdot\frac{2}{5}$

$=\frac{6}{20}$

$=\frac{3\cdot 2}{10\cdot 2}$

$=\boxed{\frac{3}{10}}$