**Pre-AP Algebra II**
**Notes Day #3**
**The Field Axioms of Algebra**

**An <u>Axiom</u> is a statement that is taken to be true to serve as a starting point for further reasoning and arguments. All branches of mathematics start with some set of axioms. In Euclidean Geometry which you just studied, you may have heard of "Euclid's five postulates". These were the axioms from which all of the geometry you studied were built.**

**Algebra has its essential axioms as well, sometimes referred to as the properties for real numbers. You should note that these axioms are what is generally referred to as satisfying the properties for a mathematical field.**

**For any real numbers, *a*, *b*, and *c*.**

| Property | Addition | Multiplication |
|---|---|---|
| **Commutative** | $a+b=b+a$ | $a \bullet b = b \bullet a$ |
| **Associative** | $(a+b)+c=a+(b+c)$ | $(a \bullet b) \bullet c = a \bullet (b \bullet c)$ |
| **Identity** | $a+0=a$ | $a \bullet 1 = a$ |
| **Inverse** | $a+(-a)=0$ | $a \bullet \frac{1}{a} = 1$, $a \neq 0$ |
| **Closure** | $a+b$ is a real number | $a \bullet b$ is a real number |
| **Distributive** | $a(b+c)=a \bullet b + a \bullet c$ | |

**Directions:** **Write a short description summarizing each of the above properties in words:**

**Commutative -** The order in which operands are placed does not change the result

**Associative -** The order in which addition/multiplication is grouped does not matter

**Identity -** Adding 0 or multiplying by 1 does not change value

**Inverse -** The inverse of addition sums to zero, inverse of multiplication has a product of 1

**Closure -** The sum or product of two real numbers is a real number

**Distributive -** Lets you multiply each addend separately and then add the products

**Directions:** **Solve the following problems and identify the property used.**

**Ex. 1:** $3+(-3)=$ 0
additive inverse

**Ex. 2:** $3 \bullet \frac{1}{3} =$ 1
multiplicative inverse

**In your own words, describe the difference of the above two examples:**

The result of the additive inverse is 0, the result of the multiplicative inverse is 1.

**Directions: Explain the error in the following example.**

**Ex. 3:** $3 \bullet \frac{1}{3} = 0$ Applies the result of the additive inverse property instead of the multiplicative inverse property.

**Example 1 was an example of the** additive inverse **property.**

**Example 2 was an example of the** multiplicative inverse **property.**

**The distinction between these two properties is important when we use the word "cancellation". It is generally assumed that when we say something "cancels" in any math problem that you can tell which of the two properties is being applied from the context of the problem.**

**Before we move on, let us define what we mean by the word "inverse".**

**Inverse (operation) -** The operation that reverses the effect of another operation

**If you look at the table on page 1 you may notice that the arithmetic operations of subtraction and division are not explicitly listed. It is common for this to be the case when looking at mathematical texts. That is because subtraction is the** additive **inverse and division is the** multiplicative **inverse.**

**Another way to state this is that subtraction is adding a negative and division is multiplying by a fraction. This is why when we perform the order of operations that addition and subtraction are interchangeable, because they are the same thing. Likewise, this follows for multiplication and division. At some point, you should become comfortable with the idea that subtraction is a specific form of addition and that division is a specific form of multiplication.**

**Directions: Rewrite Example 4 using the associative property and Example 5 using the commutative property.**

**Ex. 4:** $5 \bullet (4 \bullet 13) =$ $(5 \cdot 4) \cdot 13$

**Ex. 5:** $5 \bullet 4 \bullet 13 =$ $4 \cdot 5 \cdot 13$

**The closure and identity properties are sufficiently covered for now by our word descriptions on the previous page.**

**The final property is the distributive property, which is the link between the two fundamental operations of addition and multiplication. This property warrants a specific and detailed examination on its own at a later time.**